remainder theorm

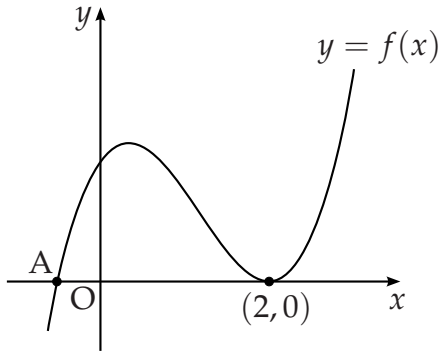
[SQA] 1. (*a*) Given that $x+2$ is a factor of $2x^3+x^2+kx+2$, find the value of k. **3**

(*b*) Hence solve the equation $2x^3+x^2+kx+2=0$ when k takes this value. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	CN	A21	$k=-5$	2001 P2 Q1
(*b*)	2	C	CN	A22	$x=-2,\frac{1}{2},1$	

•[1] ss: use synth division or f(evaluation) •[2] pd: process •[3] pd: process •[4] ss: find a quadratic factor •[5] pd: process	•[1] $f(-2)=2(-2)^3+\cdots$ •[2] $2(-2)^3+(-2)^2-2k+2$ •[3] $k=-5$ •[4] $2x^2-3x+1$ **or** $2x^2+3x-2$ **or** x^2+x-2 •[5] $(2x-1)(x-1)$ **or** $(2x-1)(x+2)$ **or** $(x+2)(x-1)$ **and** $x=-2,\frac{1}{2},1$

[SQA] 2. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(*a*) Find the x-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	NC	C8	$x = \frac{1}{3}$	2002 P2 Q3
(*b*)	3	C	NC	A21	$(x-2)(2x+1)(x-2)$	
(*c*)	2	C	NC	A6	$A(-\frac{1}{2}, 0)$, $x < -\frac{1}{2}$	

$\bullet^1$ ss: know to differentiate
$\bullet^2$ pd: differentiate
$\bullet^3$ ss: know to set derivative to zero
$\bullet^4$ pd: start solving process of equation
$\bullet^5$ pd: complete solving process

$\bullet^6$ ss: strategy for cubic, e.g. synth. division
$\bullet^7$ ic: extract quadratic factor
$\bullet^8$ pd: complete the cubic factorisation

$\bullet^9$ ic: interpret the factors
$\bullet^{10}$ ic: interpret the diagram

$\bullet^1$ $f'(x) = \ldots$
$\bullet^2$ $6x^2 - 14x + 4$
$\bullet^3$ $6x^2 - 14x + 4 = 0$
$\bullet^4$ $(3x-1)(x-2)$
$\bullet^5$ $x = \frac{1}{3}$

$\bullet^6$

$\ldots$	2	-7	4	4
		$\ldots$	$\ldots$	$\ldots$
	$\ldots$	$\ldots$	$\ldots$	0

$\bullet^7$ $2x^2 - 3x - 2$
$\bullet^8$ $(x-2)(2x+1)(x-2)$

$\bullet^9$ $A(-\frac{1}{2}, 0)$
$\bullet^{10}$ $x < -\frac{1}{2}$

3. Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(*a*) Find $g(f(x))$. **2**

(*b*) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

(*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. **5**

(*d*) Hence solve $g(f(x)) + xh(x) = 0$. **1**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	2	C	CN	A4	$3(x^3 - 1) + 1$	2011 P2 Q2
(*b*)	1	C	CN	A6	proof	
(*c*)	5	C	CN	A21	$(x - 1)(3x + 1)(x + 2)$	
(*d*)	1	C	CN	A22	$-2, -\frac{1}{3}, 1$	

$\bullet^1$ ic: interpret notation	$\bullet^1$ $g(x^3 - 1)$
$\bullet^2$ ic: complete process	$\bullet^2$ $3(x^3 - 1) + 1$
$\bullet^3$ ic: substitute and complete	$\bullet^3$ $3(x^3 - 1) + 1 + x(4x - 5)$ $= 3x^3 + 4x^2 - 5x - 2$
$\bullet^4$ ss: know to use $x = 1$	$\bullet^4$ evaluating at $x = 1$...
$\bullet^5$ pd: complete evaluation	$\bullet^5$ $3 + 4 - 5 - 2 = 0$
$\bullet^6$ ic: state conclusion	$\bullet^6$ $(x - 1)$ is a factor
$\bullet^7$ ic: find quadratic factor	$\bullet^7$ $(x - 1)(2x^2 + 7x + 2)$
$\bullet^8$ pd: factorise completely	$\bullet^8$ $(x - 1)(3x + 1)(x + 2)$
$\bullet^9$ ic: interpret and solve equation in (d)	$\bullet^9$ $-2, -\frac{1}{3}, 1$

4. (*a*) (i) Show that $(x-1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

(ii) Hence factorise $f(x)$ fully. **5**

(*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$. **1**

(*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

Find the coordinates of G. **5**

(*d*) This tangent meets the curve again at the point H.

Write down the coordinates of H. **1**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	CN	A21	$(x-1)(x-1)(2x+5)$	2010 P1 Q22
(*b*)	1	C	CN	A22	$x = 1, -\frac{5}{2}$	
(*c*)	5	C	CN	A23	$(1, -1)$	
(*d*)	1	C	CN	A23	$(-\frac{5}{2}, -8)$	

$\bullet^1$ ss: know to use $x = 1$	$\bullet^1$ evaluating at $x = 1$...
$\bullet^2$ ic: complete evaluation	$\bullet^2$ $2 + 1 - 8 + 5 = 0$
$\bullet^3$ ic: state conclusion	$\bullet^3$ $(x-1)$ is a factor
$\bullet^4$ pd: find quadratic factor	$\bullet^4$ $(x-1)(2x^2 + 3x - 5)$
$\bullet^5$ pd: factorise completely	$\bullet^5$ $(x-1)(x-1)(2x+5)$
$\bullet^6$ ic: state solutions	$\bullet^6$ $x = 1$ and $x = -\frac{5}{2}$
$\bullet^7$ ss: set $y_{\text{curve}} = y_{\text{line}}$	$\bullet^7$ $2x^3 + x^2 - 6x + 2 = 2x - 3$
$\bullet^8$ ic: express in standard form	$\bullet^8$ $2x^3 + x^2 - 8x + 5 = 0$
$\bullet^9$ ss: compare with (a) or factorise	$\bullet^9$ $(x-1)(x-1)(2x+5) = 0$
$\bullet^{10}$ ic: identify x_G	$\bullet^{10}$ $x = 1$
$\bullet^{11}$ pd: evaluate y_G	$\bullet^{11}$ $y = -1$
$\bullet^{12}$ pd: state solution	$\bullet^{12}$ $(-\frac{5}{2}, -8)$

[SQA] 5. Factorise fully $2x^3 + 5x^2 - 4x - 3$. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	4	C	NC	A21		1989 P1 Q2

$\bullet^1$ *strat*: make 2 trial divisions or 2 trial evaluations

$\bullet^2$ first linear factor

$\bullet^3$ quadratic factor

$\bullet^4$ other linear factors

$(x-1)(2x+1)(x+3)$

[SQA] 6. (*a*) Show that $x = 2$ is a root of the equation $2x^3 + x^2 - 13x + 6 = 0$. **1**

(*b*) Hence find the other roots. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	1	C	NC	A21		1999 P1 Q1
(*b*)	3	C	NC	A21		

•1 $f(2) = 16 + 4 - 26 + 6 = 0$
or
the appearance of a '0' at the end of the 3rd line in the table below

•2

2	2	1	−13	6
		4	10	−6
	2	5	−3	0

•3 $2x^2 + 5x - 3$

•4 $-3, \frac{1}{2}$

[SQA] 7. Find p if $(x + 3)$ is a factor of $x^3 - x^2 + px + 15$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A21		1990 P1 Q1

•1 *strat*: e.g. find $f(-3)$

•2 $f(-3) = 0$

•3 $p = -7$

[SQA] 8. When $f(x) = 2x^4 - x^3 + px^2 + qx + 12$ is divided by $(x - 2)$, the remainder is 114.

One factor of $f(x)$ is $(x + 1)$.

Find the values of p and q. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	C	CN	A21		1991 P1 Q6

•1 $f(2) = 114$

•2 $f(-1) = 0$

•3 $4p + 2q = 78$

•4 $p - q = -15$

•5 $p = 8, q = 23$

[SQA] 9. Find k if $x-2$ is a factor of $x^3+kx^2-4x-12$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A21		1992 P1 Q3

•1	$f(2)=8+4k-8-12$	•1	correct use of division
•2	$f(2)=0$	•2	remainder $=4k-12$
•3	$k=3$	•3	$k=3$

[SQA] 10. One root of the equation $2x^3-3x^2+px+30=0$ is -3.

Find the value of p and the other roots. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	4	C	NC	A21		1993 P1 Q7

•1 $f(-3)=-54-27-3p+30$ **or** synth. division e.g.

−3	2	−3	p	30
		−6	27	$-3p-81$
	2	−9	$p+27$	$-3p-51$

•2 $p=-17$

•3 $2x^2-9x+10$ **and** $-3p-51=0$

•4 $2, \frac{5}{2}$

[SQA] 11. (*a*) Show that $(x-3)$ is a factor of $f(x)$ where $f(x)=2x^3+3x^2-23x-12$. **2**

(*b*) Hence express $f(x)$ in its fully factorised form. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	2	C	NC	A21		1995 P1 Q2
(*b*)	2	C	NC	A21		

•1 $f(3)=2\times3^3+3\times3^2-23\times3-12$ | or equivalent division

•2 $=0$

•3 $2x^2+9x+4$

•4 $(x-3)(2x+1)(x+4)$

[SQA] 12. Express $x^4 - x$ in its fully factorised form. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	4	C	NC	A21		1996 P1 Q7

•1	$x\left(x^3-1\right)$	**OR**	•1 synthetic division or eval. $f(k)$
•2	synthetic division or eval. $f(k)$		•2 linear factor $=(x-1)$
•3	linear factor $=(x-1)$		•3 cubic factor $=\left(x^3+x^2+x\right)$
•4	$x(x-1)\left(x^2+x+1\right)$		•4 $x(x-1)\left(x^2+x+1\right)$

[SQA] 13. **(*a*) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$.** **2**
(*b*) Show algebraically that there are no other real roots. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	2	C	NC	A21		1997 P1 Q5
(*b*)	3	C	NC	A21		

•1	looking for $f(x) = \ldots\ldots = 0$	•3	$2x^2 + x + 4$
•2	$x = 2$ **explicitly stated**	•4	$b^2 - 4ac = 1 - 4 \times 2 \times 4$
		•5	$b^2 - 4ac < 0$ means no real roots

[SQA] 14. Express $x^3 - 4x^2 - 7x + 10$ in its fully factorised form. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	4	C	NC	A21		1998 P1 Q2

•1	evaluating $f(k)$ for any integer by any method	•3	quad factor *e.g.* $x^2 - 3x - 10$
•2	find 1 value of k s.t. $f(k) = 0$ *e.g.* $f(1)$ *or* $f(-2)$ *or* $f(5)$	•4	$(x-1)(x+2)(x-5)$

[SQA] 15.

(*a*) The function f is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.
The function g is defined by $g(x) = x - 1$.
Show that $f(g(x)) = x^3 - 5x^2 + 2x + 8$. **4**

(*b*) Factorise fully $f(g(x))$. **3**

(*c*) The function k is such that $k(x) = \dfrac{1}{f(g(x))}$.
For what values of x is the function k not defined? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	4	C	NC	A4		1990 P2 Q6
(*b*)	3	C	NC	A21		
(*c*)	2	C	NC	A1		

(*a*) •1 $f(g(x)) = f(x-1)$

•2 $(x-1)^3 - 2(x-1)^2 - 5(x-1) + 6$

•3 $(x-1)^3 = x^3 - 3x^2 + 3x - 1$

•4 $-2x^2 + 4x - 2 - 5x + 5 + 6$ and completing argument

(*b*) •5 *first "0" e.g.*

2	1	−5	2	8
		2	−6	−8
	1	−3	−4	0

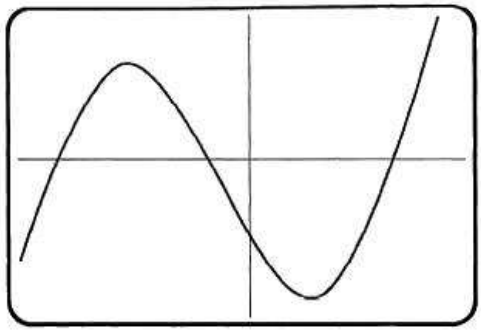
•6 $x^2 - 3x - 4 = (x+1)(x-4)$

•7 $(x-2)(x+1)(x-4)$

(*c*) •8 denominator $(= (x-2)(x+1)(x-4)) \neq 0$

•9 $-1,\ 2,\ 4$

[SQA] 16. The diagram shows part of the graph of the curve with equation $f(x) = x^3 + x^2 - 16x - 16$.



(a) Factorise $f(x)$. **(3)**

(b) Write down the co-ordinates of the four points where the curve crosses the x and y axes. **(2)**

(c) Find the turning points and justify their nature. **(6)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(a)	3	C	NC	A21		1992 P2 Q1
(b)	2	C	NC	A6		
(c)	6	C	NC	C8		

(a) •1 any linear factor

•2 corresponding quadratic factor

•3 $f(x) = (x+1)(x-4)(x+4)$

(b) •4 For all 3 points on x-axis

•5 $(0,-16)$

(c) •6 $f'(x) = 3x^2 + 2x - 16$

•7 use $f'(x) = 0$

•8 $x = 2$, **and** $x = -\frac{8}{3}$

•9 $y = -36$, **and** $y = \frac{400}{27}(14.8)$

•10

	$-\frac{8}{3}^-$	$-\frac{8}{3}$	$-\frac{8}{3}^+$	2^-	2	2^+
$f'(x)$	+	0	−	−	0	+
	⋰	⋯	⋱	⋱	⋯	⋰

•11 max at $\left(-\frac{8}{3}, \frac{400}{27}\right)$, min at $(2,-36)$

[SQA] 17. The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).

(*a*) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis. **(3)**

(*b*) Find algebraically the coordinates of the other points at which the curve crosses the x-axis. **(4)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	NC	A21		1994 P2 Q1
(*b*)	4	C	NC	A21		

(a) $\bullet^1$ strategy

eg

2	2	1	−13	a
		4	10	−6
	2	5	−3	0

or $f(2) = 0 = 16 + 4 - 26 + a$

$\bullet^2$ $a = 6$

$\bullet^3$ (0,6)

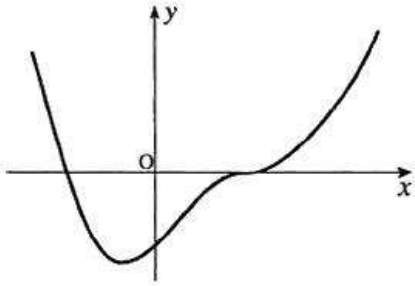
(b) $\bullet^4$ $2x^2 + 5x - 3$

$\bullet^5$ $(x+3)(2x-1)$

$\bullet^6$ $x = -3, \frac{1}{2}$

$\bullet^7$ $(-3,0), \left(\frac{1}{2},0\right)$

[SQA] 18. The function f, whose incomplete graph is shown in the diagram, is defined by $f(x) = x^4 - 2x^3 + 2x - 1$.
Find the coordinates of the stationary points and justify their nature. **(8)**



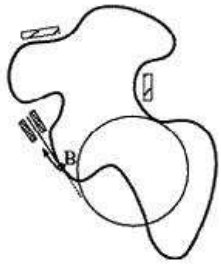
Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	8	C	CN	A21, C8		1993 P2 Q1

- $\bullet^1$ for knowing to differentiate
- $\bullet^2$ $f'(x) = 4x^3 - 6x^2 + 2$
- $\bullet^3$ for putting $f'(x) = 0$
- $\bullet^4$ for factorising **or** checking zeros
- $\bullet^5$ $x = -\frac{1}{2}$, $x = 1$
- $\bullet^6$ $y = -\frac{27}{16}$, $y = 0$
- $\bullet^7$ completed nature table

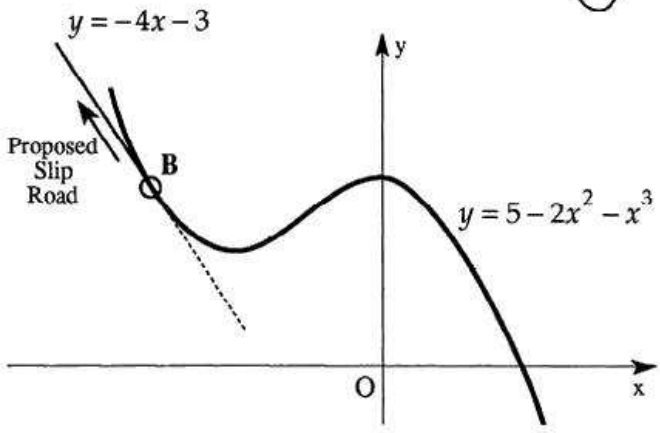
x	$< -\frac{1}{2}$	$-\frac{1}{2}$	$> -\frac{1}{2}$	< 1	1	> 1
$f'(x)$	$-ve$	0	$+ve$	$+ve$	0	$+ve$
	$\searrow$	—	$\nearrow$	$\nearrow$	—	$\nearrow$

- $\bullet^8$ $(1,0)$ is pt. of inflexion, $\left(-\frac{1}{2}, -1\frac{11}{16}\right)$ is min t.p.

[SQA] 19. The diagram shows the plans for a proposed new racing circuit. The designer wishes to introduce a slip road at B for cars wishing to exit from the circuit to go into the pits. The designer needs to ensure that the two sections of road touch at B in order that drivers may drive straight on when they leave the circuit.



Relative to appropriate axes, the part of the circuit circled above is shown below. This part of the circuit is represented by a curve with equation $y = 5 - 2x^2 - x^3$ and the proposed slip road is represented by a straight line with equation $y = -4x - 3$.



(*a*) Calculate the coordinates of B. **(7)**

(*b*) Justify the designer's decision that this direction for the slip road does allow drivers to go straight on. **(1)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	7	C	NC	A23, A21		1993 P2 Q7
(*b*)	1	A/B	NC	A24		

(*a*) •1 equating expressions for y

•2 re-arranging cubic..... "..." = 0

•3 strategy for solving cubic

•4 first linear factor

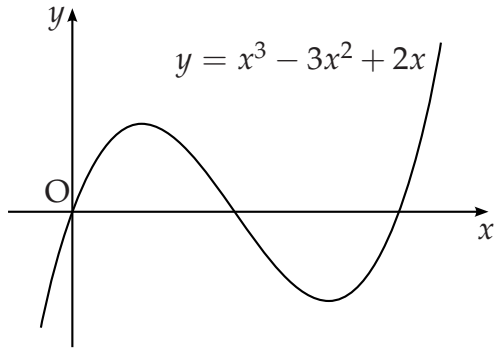
•5 quadratic factor

•6 $x = -2,\ 2$

•7 intersection at $(-2, 5)$

(*b*) •8 double root ⇒ tangency **or** $y'(-2) = -4$ = gradient of line

[SQA] 20. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	CN	C5	$x + y = 1$	2000 P2 Q1
(*b*)	5	C	CN	A23, A22, A21	$(-1, -6)$	

$\bullet^1$ ss: know to differentiate	$\bullet^1$ $y' = \ldots$
$\bullet^2$ pd: differentiate correctly	$\bullet^2$ $3x^2 - 6x + 2$
$\bullet^3$ ss: know that gradient $= f'(1)$	$\bullet^3$ $y'(1) = -1$
$\bullet^4$ ss: know that y-coord $= f(1)$	$\bullet^4$ $y(1) = 0$
$\bullet^5$ ic: state equ. of line	$\bullet^5$ $y - 0 = -1(x - 1)$
$\bullet^6$ ss: equate equations	$\bullet^6$ $2x - 4 = x^3 - 3x^2 + 2x$
$\bullet^7$ pd: arrange in standard form	$\bullet^7$ $x^3 - 3x^2 + 4 = 0$
$\bullet^8$ ss: know how to solve cubic	$\bullet^8$ $\begin{array}{c\|cccc} \ldots & 1 & -3 & 0 & 4 \\ & & \ldots & \ldots & \ldots \\ \hline & \ldots & \ldots & \ldots & \ldots \end{array}$
$\bullet^9$ pd: process	$\bullet^9$ identify $x = -1$ from working
$\bullet^{10}$ ic: interpret	$\bullet^{10}$ $(-1, -6)$

[SQA] 21.

(*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(*b*) Solve $\log_2(x+3) + \log_2(x^2 + 5x - 4) = 3$. **5**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	4	C	CN	A21	$(x-1)(x+4)(x+5)$	2009 P2 Q3
(*b*)	5	B	CN	A32	$x = 1$	

•[1] ss: know and use $f(a) = 0 \Leftrightarrow a$ is a root	•[1] $f(1) = 1 + 8 + 11 - 20 = 0$ so $x = 1$ is a root
•[2] ic: start to find quadratic factor	•[2] $(x-1)(x^2 \cdots$
•[3] ic: complete quadratic factor	•[3] $(x-1)(x^2 + 9x + 20)$
•[4] pd: factorise fully	•[4] $(x-1)(x+4)(x+5)$
•[5] ss: use log laws	•[5] $\log_2\big((x+3)(x^2+5x-4)\big)$
•[6] ss: know to & convert to exponential form	•[6] $(x+3)(x^2+5x-4) = 2^3$
•[7] ic: write cubic in standard form	•[7] $x^3 + 8x^2 + 11x - 20 = 0$
•[8] pd: solve cubic	•[8] $x = 1$ or $x = -4$ or $x = -5$
•[9] ic: interpret valid solution	•[9] $x = 1$ only

[END OF QUESTIONS]